HEAT SOURCE AND WALL EFFECTS ON PERISTALTIC FLOW OF A VISCO-ELASTIC RIVLIN–ERICKSEN FLUID

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ABSTRACT

The present paper investigates the peristaltic flow of Visco-elastic fluid in the presence of heat source with wall properties in a two-dimensional flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for stream function, temperature and heat transfer coefficient. The effects of elastic parameters and pertinent parameters on the coefficient of heat transfer have been computed numerically. It is observed that temperature (θ) reduces marginally with increase in heat source parameter, the rigidity of the wall (E_1) and the stiffness of the wall (E_2) of order 0.2 and further increase in E_1 and E_2 the depreciation in 'θ', with E_1 or E_2 is appreciably large.

Keywords: Peristaltic flow, Rivlin–Ericksen Fluid, Wall properties, Heat Source.

1. INTRODUCTION

The study of the mechanism of peristalsis, in both physiological and mechanical situations, has become the object scientific research. From fluid mechanical point of view peristaltic motion is defined as the flow of generated by a wave traveling along the walls of an elastic tube. In physiology it may be described as a progressive wave of area contraction or expansion along a length of a distensible tube containing fluid provided with transverse and muscular fibres. It consists in narrowing and transverse shortening of a portion of the tube which then relaxes while the lower portion becomes shortened and narrowed. The mechanism of peristalsis occur for urine transport from kidney to bladder through the ureter, movement of chime in the gastro-intestinal tract, the movement of spermatozoa in the ducts afferent's of the mail reproductive tract, movement of ovum in the fallopian tube, vasomotion in small blood vessels, the food mixing and motility in the intestines, blood flow in cardiac chambers etc. Also, bio-medical instruments such as heart-lung machine use peristalsis to pump blood while mechanical devices like roller pumps use this mechanism to pump and other corrosive fluids. The problem of the mechanism of peristalsis transport has attracted the attention of many investigators. Fung and Yih (1968), Shapiro and Jaffrin et al. (1969) have studied peristaltic pumping with long wavelength at low Reynolds number. Mitra et.al (1973) investigated the influence of wall properties and Poiseuillie flow in peristalsis.

The study of two-phase flows finds applications in many branches of Engineering, Environmental, Physical Sciences, etc. A few examples of such flows in diverse fields are the flow of dissolved micro molecules of fibre suspensions in paper making, flow of blood through arteries, propulsion and combustion in rockets, dispersion and fall out of pollutants in air, erosion of material due to continuous impingement of suspended particles in air etc. Frederick (1949) studied two phase fluid-solid flow. To develop a mathematical theory of blood flow in arteries, Alihasan Nayfeh (1966) considered blood as binary system of plasma (liquid phase) and blood cells a (solid phase). Saffman (1962) dusty fluid serves as a better model to describe blood as a binary system. Solid-particle motion in two-dimensional peristaltic flows has been discussed by Hung et.al (1976). Kaimal, M.R.(1978) investigated peristaltic pumping of a Newtonian fluid with particles suspended in it at low Reynolds number under long wavelength approximation. Nag, S.K. (1980) studied the two-dimensional flow of unbounded dusty fluid induced by the sinusoidal transverse motion of an infinite wall. Radhakrishnamacharya (1978) studied the pulsatile flow of a fluid containing The interaction of peristalsis and heat transfer has become highly relevant and significant in several industrial processes also thermodynamical aspects of blood become significant in process like haemodialysis and oxygenation when blood is drawn out of the body. Srinivasulu and Radhakrishnamacharya (2007) studied the influence of wall properties on peristaltic transport with heat transfer. Rajaneesh kumar., (2013) investigated numerical study of free convection heat and mass transfer in a Rivlin–Ericksen viscoelastic flow past an impulsively started vertical plate with variable temperature and concentration.

Thermo dynamical aspects of blood may not be important when blood is inside the body but they become significant when blood is drawn out of the body in processes like hemodialysis and oxygenation. Keeping these things in view, victor and shah (1925) considered heat transfer to blood flowing in a tube assuming blood to be Casson fluid. The study of interaction of peristalsis with heat transfer may lead to better understanding of the role of peristalsis and the flow phenomenon in physiological systems.

Hence, in this chapter, we have investigated the interaction of peristalsis with heat transfer for the motion of a Visco-elastic Rivlin – Erickson fluid in a two-dimensional flexible channel under heat source and wall effects. Assuming that the wavelength of the peristaltic wave is large in comparison to the mean-half width of the channel, the momentum and energy equations have been linearized and analytical solutions for stream function and temperature have been obtained in terms of the wall slope parameter. The effects of pertinent parameters on temperature and heat transfer have been studied.

2. FORMULATION OF THE PROBLEM

Consider a peristaltic flow of an incompressible visco-elastic Rivlin-Erickson fluid through a two-dimensional channel of uniform thickness. The walls of the channel are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.

We consider a peristaltic flow of a Visco-elastic fluid through two-dimensional channel of width 2d, symmetric with respect to its axis. The walls of the channel are assumed to be flexible. The travelling waves are represented by (Fig.1)

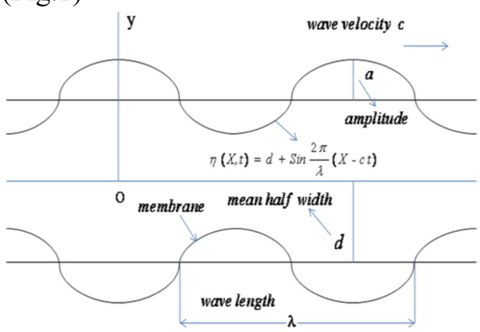

Fig.1 Geometry of the problem

The geometry of the flexible walls is represented by

$$y = \eta(x,t) = d + a\ \text{Sin}\ \frac{2\pi}{\lambda}(x - ct) \qquad (1)$$

Where ' a ' is the amplitude of the peristaltic wave, ' c ' is the wave velocity, ' λ ' is the wavelength. The equations governing the two-dimensional flow of Rivlin -Erickson fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial u}{\partial t} + v\ \frac{\partial u}{\partial y} = -\frac{1}{\rho}\ \frac{\partial p}{\partial x} + \upsilon\ \frac{\partial^2 u}{\partial y^2} + \beta\ \frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) \qquad (3)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\frac{\partial^2 v}{\partial y^2} + \beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y}\right) \tag{4}$$

Where 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'β' is the coefficient of visco-elasticity, 'υ' is the coefficient of kinematic viscosity.

The Equation of energy is given by

$$\rho C_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q(T_0 - T) + \upsilon\left(\frac{\partial u}{\partial x}\right)^2 - \beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t\,\partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) \tag{5}$$

Where 'k' is the coefficient of thermal conductivity, C_p' is the specific heat at constant pressure, 'T' is the temperature, 'Q' is the heat.

The governing equation of motion of the flexible wall may be expressed as

$$L(\eta) = p - p_0 \tag{6}$$

Where 'L' is an operator, which is used to represent the motion of stretched membrane with damping

$$L \equiv -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t} \tag{7}$$

For simplicity, we assume $P_0 = 0$.

The horizontal displacement assumed to be zero, gives the boundary conditions are

$$u = 0 \quad \text{at} \quad y = \pm\eta = \pm[d + a\,\mathrm{Sin}\frac{2\pi}{\lambda}(x - ct)] \tag{8}$$

$$\frac{\partial}{\partial x}L(\eta) = \frac{\partial p}{\partial x} = \rho\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \quad \text{at} \quad y = \pm\eta \tag{9}$$

$$T = T_0 \quad \text{on} \quad y = -\eta\,, \quad T = T_1 \quad \text{on} \quad y = \eta \tag{10}$$

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function'ψ 'such as

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x} \tag{11}$$

and introducing non-dimensional quantities

$$x^* = \frac{x}{\lambda},\ y^* = \frac{y}{d},\ u^* = \frac{u}{c},\ v^* = \frac{v}{c\delta},\ \psi^* = \frac{\psi}{cd},\ t^* = \frac{ct}{\lambda},\ \eta^* = \frac{\eta}{d},\ p^* = \frac{p^2}{\mu c\lambda}\ \theta = \frac{T - T_0}{T_1 - T_0} \tag{12}$$

In equations (1) – (5), (7) – (9) and eliminating 'p', we finally get (after dropping asterisk)

$$\delta\left\{\left(\frac{\partial}{\partial t}\left(\frac{\partial^2\psi}{\partial y^2} + \delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right) - \frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2} + \delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right) - \frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2} + \delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right)\right\} =$$

$$S\delta\left\{\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial y^4}\right) - \frac{\partial^4\psi}{\partial x\,\partial y^3}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\psi}{\partial x}\frac{\partial^5\psi}{\partial y^5}\right) + 2\delta^2\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial x^2\,\partial y^2}\right) - \frac{\partial^4\psi}{\partial x\,\partial y^3}\frac{\partial^2\psi}{\partial x^2} - \frac{\partial\psi}{\partial x}\frac{\partial^5\psi}{\partial x^2\,\partial y^3}\right)\right\}$$

$$+\frac{1}{R}\left(\frac{\partial^4\psi}{\partial y^4} + \delta^2\frac{\partial^4\psi}{\partial x^2\,\partial y^2}\right)$$

(13)

$$R\,\delta\left(\frac{\partial\theta}{\partial t}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial y^2}\right)-\alpha\,\theta+E\left(\frac{\partial^2\psi}{\partial y^2}\right)^2-S\,R\,E\,\delta\left(\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial t\,\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial y^3}\right) \quad (14)$$

$$\frac{\partial\psi}{\partial y}=0 \quad \text{at} \quad y=\pm\eta=\pm[1+\varepsilon\ \text{Sin}\ 2\pi(x-t)] \quad (15)$$

$$\left(\frac{\partial^3\psi}{\partial y^3}+\delta^2\frac{\partial^3\psi}{\partial x^2\,\partial y}\right)-R\,\delta\left(\frac{\partial^2\psi}{\partial y\,\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)$$
$$\frac{R\,\alpha}{\tau}\left(\frac{\partial\varphi}{\partial y}-\frac{\partial\psi}{\partial y}\right)=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\,\partial t^2}+E_3\frac{\partial^2}{\partial x\,\partial t}\right)\eta \quad \text{at} \quad y=\pm\eta \quad (16)$$

$$\theta=0 \quad \text{on} \quad y=-\eta, \qquad \theta=1 \quad \text{on} \quad y=\eta \quad (17)$$

Where $\varepsilon\,(=a/d)$, $\delta\,(=d/\lambda)$ are geometric parameters, $R\,(=cd/\nu)$ is the Reynolds number, $P_r(=\rho\,c_p\,\nu/k)$ is the Prandtl number, $\alpha=Q\,d^2/\rho\,c_p\nu$ is the Source parameter, $E=c^2/\rho\,c_p(T_1-T_0)$ is the Ecert number, $S=\beta/d^2$ Visco- elastic parameter, $E_1\,(=-T\,d^3/\lambda^3\rho\,\nu\,c)$, $E_2\,(=m\,c\,d^3/\lambda^3\rho\,\nu\,c)$, $E_3\,(=C\,d^3/\lambda^2\rho\,\nu\,c)$ are the non-dimensional elasticity parameters.

3. METHOD OF SOLUTION

We seek perturbation solution in terms of small parameter δ as follows:

$$F=F_0+\delta\,F_1+\delta^2\,F_2+\ldots\ldots \quad (18)$$

where F represents any flow variable.

Substituting (18) in equations (13) to (17) and collecting the coefficients of various powers of δ and solving the resultant equations under the relevant boundary conditions, we finally get

$$\psi=\psi_0+\delta\,\psi_1+\ldots\ldots \quad (19)$$

The temperature coefficient in terms of wall slope parameter ‘δ’ is

$$\theta=\theta_0+\delta\,\theta_1+\ldots\ldots \quad (20)$$

Where

$$\psi_0=A\,(y^3-3\,\eta^2 y) \quad (21)$$

$$\psi_1=R\,(B_1y^7+B_2y^5-B_3y^3+B_4y) \quad (22)$$

$$\theta_0=C_1\ \text{Cosh by}+C_2\ \text{Sinh by}+G_1y^2+G_2 \quad (23)$$

$$\theta_1=C_3\ \text{Cosh by}+C_4\ \text{Sinh by}+D_1\ y\,\text{Sinh by}+d_3\ y\,\text{Cosh by}+d_6(b^3y^4\text{Cosh by}-2\,b^2y^3\text{Sinh by}+3\,b\,y^2\ \text{Cosh by}-3\,y\,\text{Sinh by})+D_2(b\,y^2\ \text{Cosh by}-y\,\text{Sinh by})+D_3(b\,y^2\ \text{Sinh by}-y\,\text{Cosh by})+d_6(b^3y^4\text{Sinh by}-2\,b^2y^3\text{Cosh by}+3\,b\,y^2\ \text{Sinh by}-3\,y\,\text{Cosh by})-D_4(b^4\,y^4+12\,b^2\,y^2+24)-d_{15}(b^6y^6+30\,b^4y^4+360\,b^2y^2+720)-D_5(b^2\,y^2+2)-d_5$$
(24)

The heat transfer coefficient in terms of wall slope parameter ‘δ’ is

$$z=z_0+\delta\,z_1+\ldots\ldots \quad (25)$$

$$z_0 = \left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial \theta_0}{\partial y}\right) \tag{26}$$

$$z_1 = \left(\frac{\partial \theta_0}{\partial x}\right) + \left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial \theta_1}{\partial y}\right) \tag{27}$$

All the constants depend upon parameters.

4. RESULTS AND DISCUSSION

The non-dimensional temperature distribution 'θ' is depicted in figs.(2-7) for a different parametric value of E_1, E_2, E_3, p_r, E and α. From figs. (2-3) represents of the variation 'θ' with increasing E_1 and E_2, we noticed that 'θ' reduces marginally with increase in E_1 and E_2 of order 0.2 and further increase in E_1 and E_2 the depreciation in 'θ', with E_1 or E_2 is appreciably large, while an increase the elastic parameter E_3 enhances 'θ' marginally in the entire region fig.(4). In fig. (5), we find that 'θ' experiences depreciation with increase in the Prandtl number p_r. The effect of dissipation of 'θ' is exhibited in fig. (6) is evident from this figure. The inclusion of dissipation term reduces the temperature marginally in the flow region except in the region $0.6 \leq y \leq 0.9$. From fig. (7) represents of the variation 'θ' with increase in α, we noticed that 'θ' reduces marginally with increase in α of order 0.2 and further increase in α the depreciation in 'θ', with α is appreciably large.

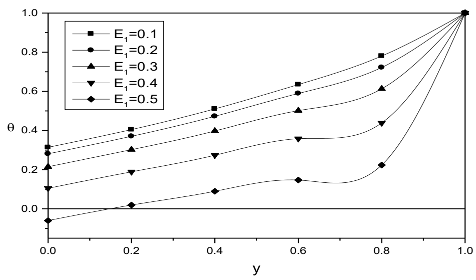

Fig.2-Effect of $\mathbf{E_1}$ on $\boldsymbol{\theta}$
(ε=0.01, δ=0.01, R=0.1, E_2=0.2, E_3=0.3, α =1, S=1, E=1, p_r =1)

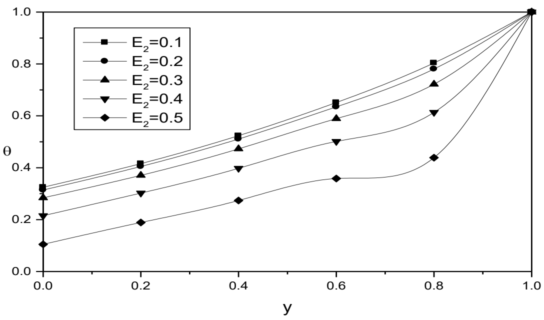

Fig.3-Effect of $\mathbf{E_2}$ on $\boldsymbol{\theta}$
(ε=0.01, δ=0.01, R=0.1, E_1=0.1, E_3=0.3, α =1, S=1, E=1, p_r =1)

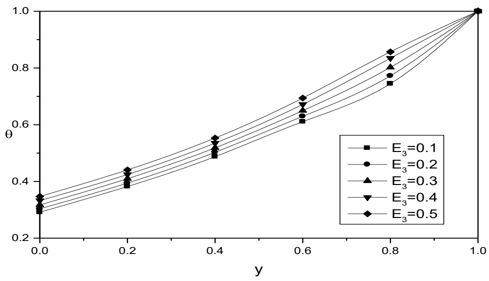

Fig.4-Effect of $\mathbf{E_3}$ on $\boldsymbol{\theta}$
(ε=0.01, δ=0.01, R=0.1, E_1=0.1, E_2=0.2, α =1, S=1, E=1, p_r =1)

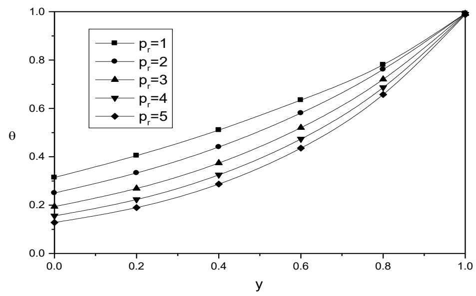

Fig.5-Effect of p_r on $\boldsymbol{\theta}$
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =1, S=1, E=1)

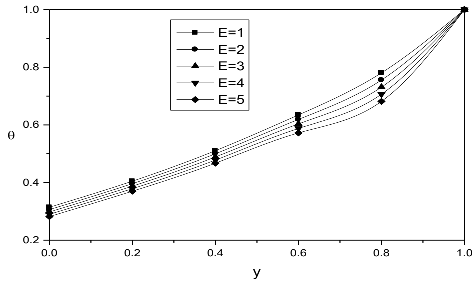

Fig.6-Effect of **E** on **θ**
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =1, S=1, p_r =1)

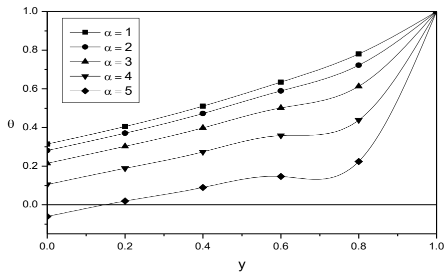

Fig.7-Effect of α on **θ**
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, E =1, S=1, p_r =1)

Figs.(8-13) represents the variance of Heat Transfer coefficient z with E_1, E_2, E_3, E, p_r and α. An increase in Elasticity parameters E_1, E_2 through smaller values enhances marginally in the entire flow region, but for higher for E_1 & $E_2 \geq 0.3$, a valuation in z is large and its increment for $E_1 = E_2 = E_3 = 0.5$ is remarkably large comparative with other values fig.(8-9),while the variation of z with E_3 shows that the variation of z is uniform for all values of E_3 fig.(10). The effect of dissipation on the Heat Transfer coefficient z is shown in fig.(11), we observe that the inclusion of dissipation term in energy equation leads to an enhancement in z. Also, an increase in the Prandtl number 'p_r' results in a depreciation in the z. We notice that for smaller values of $p_r \leq 0.8$ the depreciation in z is remarkable and further higher p_r, the depreciation is marginal in everywhere in the flow region fig.(12). An increase in α, z enhances marginally in the entire flow region, but for higher for $\alpha \geq 0.3$, a valuation in z is large and its increment for α = 0.5 is remarkably large comparative with other values fig.(13).

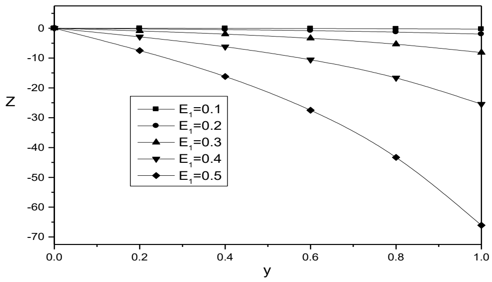

Fig.8-Effect of **E_1** on **Z**
(ε=0.01, δ=0.01, R=0.1, E_2=0.2, E_3=0.3, α =1, S=1, E=1, p_r =1)

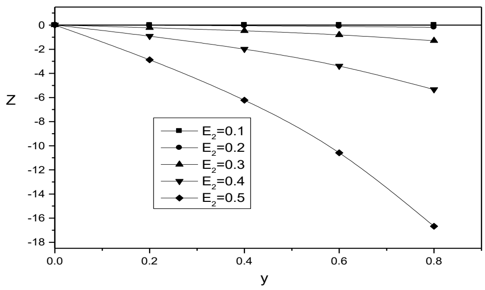

Fig.9-Effect of **E_2** on **Z**
(ε=0.01, δ=0.01, R=0.1, E_1=0.1, E_3=0.3, α =1, S=1, E=1, p_r =1)

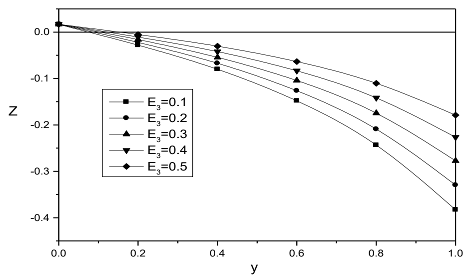

Fig.10-Effect of **E_3** on **Z**
(ε=0.01, δ=0.01, R=0.1, E_1=0.1, E_2=0.2, α =1, S=1, E=1, P=1)

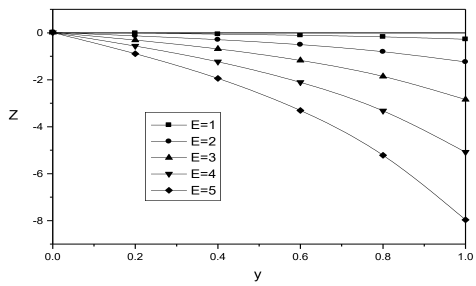

Fig.11-Effect of **E** on **Z**
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =1, S=1, p_r =1)

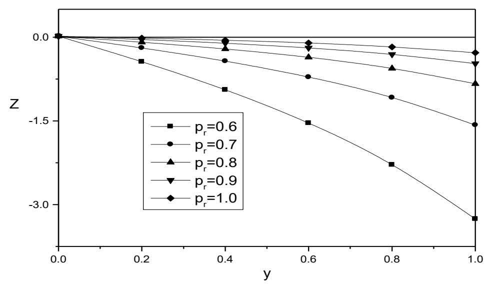

Fig.12-Effect of $\mathbf{p_r}$ on **Z**
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =1, S=1, E=1)

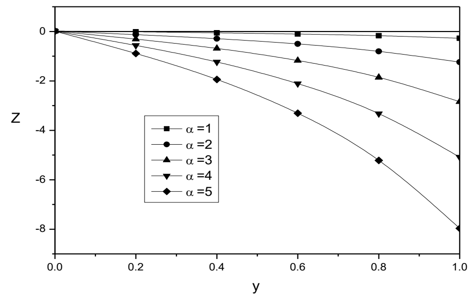

Fig.13-Effect of α on **Z**
(R=0.1, ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, S =1, p_r =1, E=1)

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